MetaMath: Bootstrap Your Own Mathematical Questions for Large Language Models

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Methods

Answer Augmentation (AnsAug)

Question: What is the total amount that James paid when he purchased 5 packs of beef, each weighing 4 pounds, at a price of $5.50 per pound?
Answer: (sample answers from GPT-3.5-Turbo)

Rephrasing Question

Question: What is the total amount that James paid when he purchased 5 packs of beef, each weighing 4 pounds, at a price of $5.50 per pound?
Answer: Each pack of beef weighs 4 pounds, so 5 packs weigh 4 * 5 = 20 pounds in total. The price per pound of beef is $5.50, so the total cost for 20 pounds is 20 * $5.50 = $110. ... The answer is: 110.

Backward Question by Self-Verification (needs rewriting) (EMNLP 2023)

Question: James buys x packs of beef that are 4 pounds each. The price of beef is $5.50 per pound. He paid 110. What is the value of unknown variable x?
Answer: To solve this problem, we need to determine the value of x, which represents the number of packs of beef that James bought. Each pack of beef weighs 4 pounds and ... The value of x is 5.

Backward Question by FOBAR (Preprint 2023)

Question: James buys x packs of beef that are 4 pounds each. The price of beef is $5.50 per pound. How much did he pay? If we know the answer to the above question is 110, what is the value of unknown variable x?
Answer: James buys x packs of beef that are 4 pounds each, so he buys a total of 4x pounds of beef. The price of beef is $5.50 per pound, so the total cost of the beef is 5.50 * 4x = 22x. ... The value of x is 5.

Datasets: *395K augmented samples by 4 methods on original GSM8K & MATH training data*

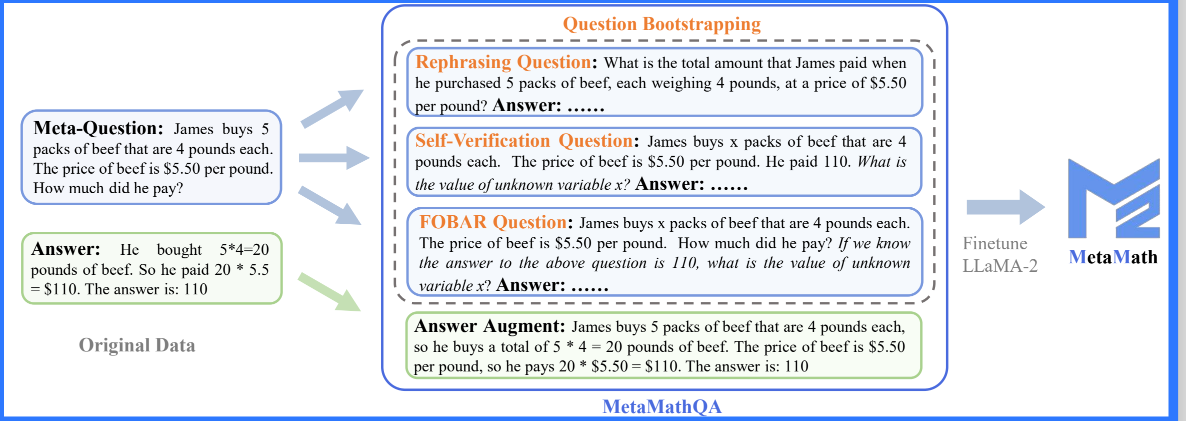
Dataset	AnsAug	Rephrasing	SV	FOBAR	Overall
MetaMathQA-GSM8K	80K	80K	40K	40K	240K
MetaMathQA-MATH	75K	50K	15K	15K	155K
MetaMathQA	155K	130K	55K	55K	395K

Ablation: *data augmented by 4 methods all have performance gains.*

Method	GSM8K						MATH					
	AnsAug	Rep.	SV	FOBAR	GSM8K	MATH	AnsAug	Rep.	SV	FOBAR	GSM8K	MATH
SFT	✗	✗	✗	✗	41.6	3.0	✗	✗	✗	✗	13.8	4.7
MetaMath	✓	✗	✗	✗	59.6	4.4	✓	✗	✗	✗	28.4	12.9
	✗	✓	✗	✗	59.7	4.4	✗	✓	✗	✗	30.4	12.4
	✓	✓	✗	✗	60.6	4.4	✓	✓	✗	✗	29.1	15.3
	✓	✓	✓	✓	**64.4**	**5.7**	✓	✓	✓	✓	**34.6**	**17.7**

Spotlight

We improve LLM's **Math** reasoning ability in both **forward reasoning** & **backward reasoning**



Meta-Question: James buys 5 packs of beef that are 4 pounds each. The price of beef is $5.50 per pound. How much did he pay?

Answer: He bought 5*4=20 pounds of beef. So he paid 20 * 5.5 = $110. The answer is: 110

Original Data

Question Bootstrapping

Rephrasing Question: What is the total amount that James paid when he purchased 5 packs of beef, each weighing 4 pounds, at a price of $5.50 per pound? **Answer: ……**

Self-Verification Question: James buys x packs of beef that are 4 pounds each. The price of beef is $5.50 per pound. He paid 110. *What is the value of unknown variable x?* **Answer: ……**

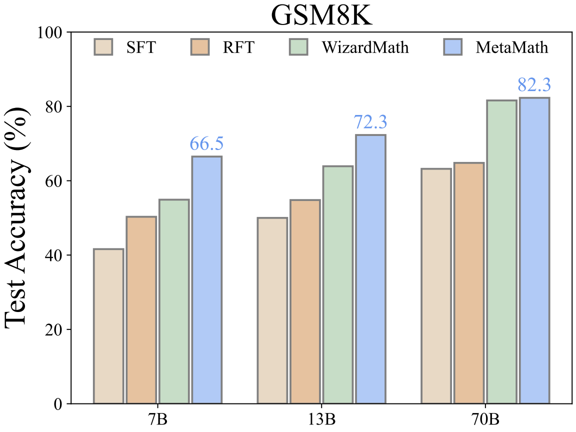
FOBAR Question: James buys x packs of beef that are 4 pounds each. The price of beef is $5.50 per pound. How much did he pay? *If we know the answer to the above question is 110, what is the value of unknown variable x?* **Answer: ……**

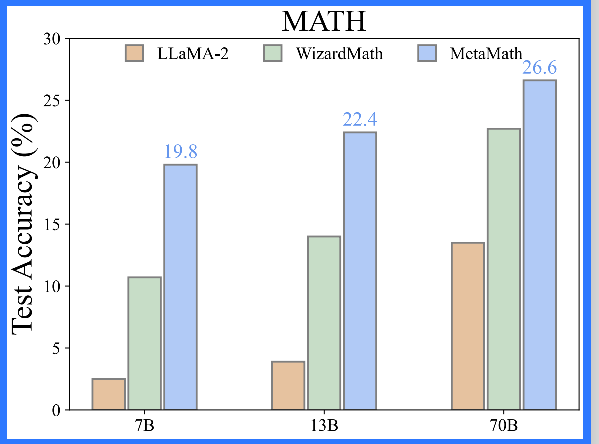
Answer Augment: James buys 5 packs of beef that are 4 pounds each, so he buys a total of 5 * 4 = 20 pounds of beef. The price of beef is $5.50 per pound, so he pays 20 * $5.50 = $110. The answer is: 110

MetaMathQA

Finetune LLaMA-2

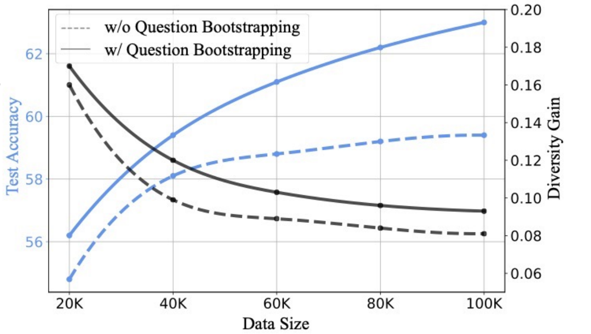
MetaMath



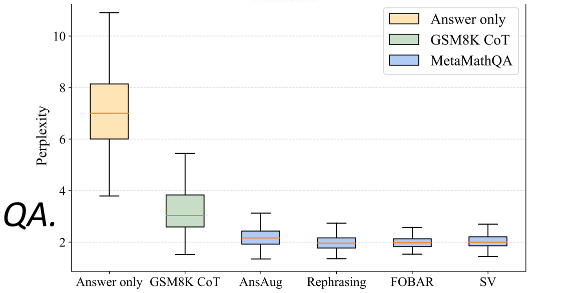


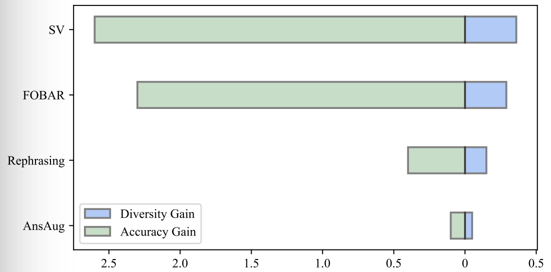
Scan to
code / data / checkpoints

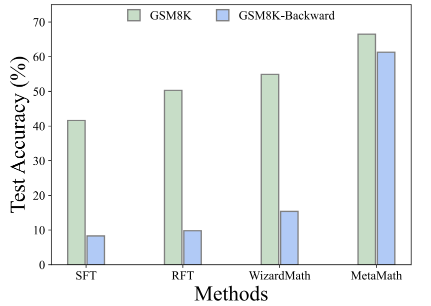
Diversity & Accuracy: *Naïve data augmentation suffers a quick saturation in accuracy. Thanks to high diversity, MetaMath alleviates saturation.*



Lower perplexity of MetaMathQA: *Pretrained models (e.g. LLaMA-2) have lower perplexity on MetaMathQA.*







Left: *SV & FOBAR bring higher diversity, resulting in higher performance gains.* ***Right:*** *MetaMath achieves better backward reasoning ability than existing methods.*

MetaMath on OOD tasks & stronger models:

Performance on DROP dataset

Different from GSM8K & MATH, questions in DROP have longer reasoning context. MetaMath performs better than baselines.

	#Params	Accuracy (Exact Match)
SFT	7B	25.8
RFT	7B	26.7
WizardMath	7B	31.5
MetaMath	7B	**37.1**
WizardMath	13B	46.4
MetaMath	13B	**49.5**
WizardMath	70B	63.1
MetaMath	70B	**72.3**

Performance on Llemma & Mistral

On stronger models such as Llemma & Mistral, Metamath also boosts performance.

	MetaMathQA	GSM8K	MATH
LLaMA-2-7B	✗	14.6	2.5
	✓	**66.5**	**19.8**
LLaMA-2-13B	✗	28.7	3.9
	✓	**72.3**	**22.4**
Llemma-7B	✗	36.4	18.0
	✓	**69.2**	**30.0**
Mistral-7B	✗	52.2	13.1
	✓	**77.7**	**28.2**